IMPROVED EQUIVALENT CIRCUIT AND FINITE ELEMENT METHOD MODELING OF CAPACITIVE MICROMACHINED ULTRASONIC TRANSDUCERS

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Abstract — **Equivalent circuit model has been widely used to predict the bandwidth of capacitive micromachined ultrasonic transducers (CMUTs). According to this model, the lower cutoff of the bandwidth is determined by the time constant of the parallel RC where R is dictated by the radiation and C is determined by the electrical capacitance of the transducer. The higher cutoff, on the other hand, is determined by the membrane's anti-resonance. In the mechanical part of the model, the radiation impedance is simply added to the membrane impedance assuming that the membrane impedance does not change when it operates in the immersion medium. Therefore, the mass loading effect of the medium is neglected. Our finite element method calculations showed that the mass loading on the membrane impedance drastically lowers the membrane anti-resonance frequency degrading the bandwidth. In this paper, we present results of equivalent circuit modeling combined with finite element analysis. We constructed a 3D finite element model for one element of a 1D array. The element has 7 hexagonal membranes in the width dimension and it is assumed that the membranes are replicated in the length dimension infinitely by using symmetry boundary conditions. By combining membrane impedance with equivalent circuit model, we found that the center frequency of operation is 11 MHz and the bandwidth is 12.5 MHz close to the collapse voltage. We also investigated the effect of the DC bias on the center frequency. Decreasing the bias voltage increased the center frequency without affecting the bandwidth assuming the source impedance is zero.**

I. INTRODUCTION

The analogy between mechanical structures and electrical circuits is widely used for the analysis of mechanical systems. Using the analogy, forces are replaced by voltage sources and velocities are replaced by electrical currents. Then, an equivalent circuit of the system is constructed. This method becomes an even more powerful tool for the analysis of electromechanical systems where some parts of the system are already in electrical domain. For example, equivalent circuit analysis was successfully employed for piezoelectric transducers for their design and optimization [1]. Similarly, Hunt used the same approach for the modeling of electrostatic transducers [2]. Recently, the equivalent circuit model is employed for the characterization of capacitive micromachined ultrasonic transducers (CMUTs) [3,4]. Fig. 1 shows the equivalent circuit for a CMUT transducer. In the electrical part, C_o is the clamped capacitance of the device. Spring softening capacitance and the mechanical membrane impedance constitute the mechanical part. The two parts are coupled together through an electromechanical transformer. For a parallel plate capacitor, electrical capacitance, electrical field and conversion ratio are given by

$$C_0 = \frac{\varepsilon_o S}{g - x} \quad , \quad E_{em} = \frac{V}{g - x} \quad , \quad n = C_0 E_{em} \qquad (1)$$

where S is the area, g is the initial gap, x is the membrane displacement and V is the bias voltage.

When the transducer is operated in vacuum, the mechanical port of the circuit is short-circuited. For immersion devices, the mechanical port is simply terminated by the radiation impedance.

In this paper, we will test the validity of the equivalent circuit model by using finite element method (FEM). First, the results of input impedance calculations of the transducer in vacuum will be compared using the two methods to find accurate values for the circuit elements. This analysis will include the membrane deflection. Then, the bandwidth of the device will be evaluated for immersion operation.

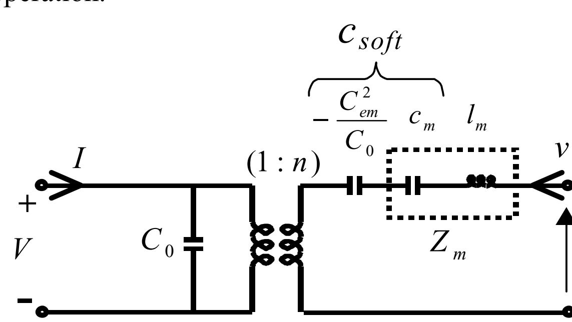

Figure 1. Electrical equivalent circuit of a CMUT.

II. FINITE ELEMENT METHOD CALCULATIONS

We used ANSYS 5.7 for the FEM calculations. In the following subsections, 2D and 3D models will be developed for vacuum and immersion devices. For immersion devices, the bandwidth of the transducer will be evaluated using both circuit model and FEM.

A. Input impedance in vacuum

Fig. 2 depicts the finite element model that was used in the calculations. The model was assumed to be axisymmetric around the line passing through the membrane center. The membrane was meshed by plane elements and the gap was meshed by electrostatic parallel plate capacitors (C_i) as shown in the figure. The clamped capacitance and turns ratio are then given by

$$C_0 = \sum C_i \quad , \quad n = \sum C_i E_i \qquad (2)$$

where C_i and E_i are the capacitance and electric field in the small capacitor elements, respectively. By using this model first, we calculated the input impedance of a membrane that moves parallel to the substrate. We simply coupled the movement of the nodes on the electrode together in the vertical direction. This structure resulted in a collapse voltage of 199.6 V. Assuming 199 V bias voltage, the C_o and n found to be 13.84 fF and 1.3x10^{-5} nt/V, respectively. The electrical input impedance is given by

$$Z_{in} = \frac{V_{AC}}{\sum I_{cap}} \qquad (3)$$

Fig. 3 shows the input impedance calculated using FEM and equivalent circuit for a parallel plate membrane. Results obtained using the two methods agree very well.

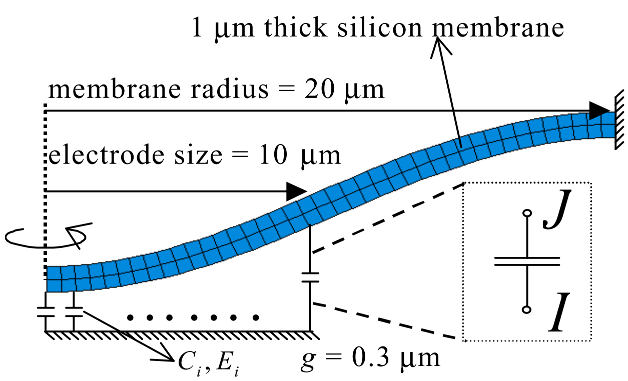

Figure 2. Finite element model for vacuum calculations

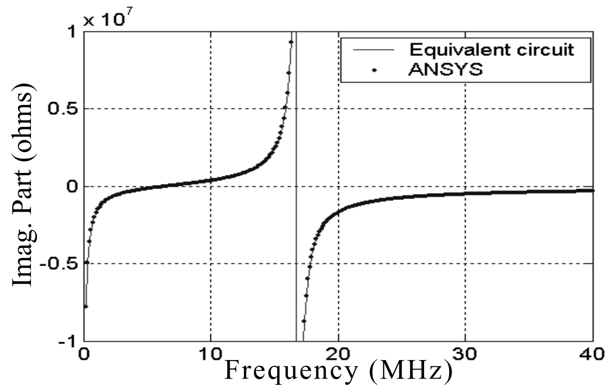

Figure 3. Input impedance for a parallel plate capacitor.

After testing the FEM model on a parallel plate capacitor, we released the coupling over the electrode region. For a membrane motion, the device capacitance can be divided into two parts: a parallel plate capacitance and parasitic capacitance [5]. This evaluation can be carried out through the electromechanical coupling coefficient calculation. For more accurate circuit modeling this parasitic capacitance should be included. Our calculations showed that for the geometry depicted in Fig. 2, the parasitic capacitance is 8%. Another difficulty arises due to the calculation of turns ratio in (2). The electrical forces are only applied to the electrode region. However, the membrane impedance is calculated as the ratio of the uniform applied force all over the membrane to the average membrane velocity. To solve this discrepancy, one needs to define an effective n^* through

$$n^* = n \frac{Z_m}{Z_p} \qquad (4)$$

where Z_m is the membrane impedance and Z_p is the partial impedance calculated assuming a uniform force over the electrode. We calculated Z_m and Z_p using FEM and for the geometry in Fig.2, the effective n^* is

$$n^* = 2.3n \qquad (5)$$

If the above improvements are incorporated in the circuit model one can obtain a very good match between the circuit model and FEM calculations as shown in Fig.4.

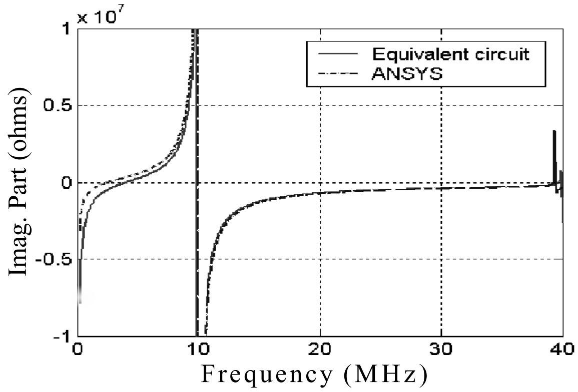

Figure 4. Input impedance using improved circuit model and FEM.

B. *Immersion FEM calculation (single cell)*

For immersion devices, the mechanical port of the circuit model is terminated by the radiation impedance of the medium. Therefore, the impedance seen by the membrane is given by the summation of the mechanical impedance of the membrane in vacuum and the radiation impedance. In this subsection, we will test the validity of this approach for a single membrane by calculating its mechanical impedance in the immersion medium.

Fig. 5 shows the FEM model of a single membrane immersed in water.

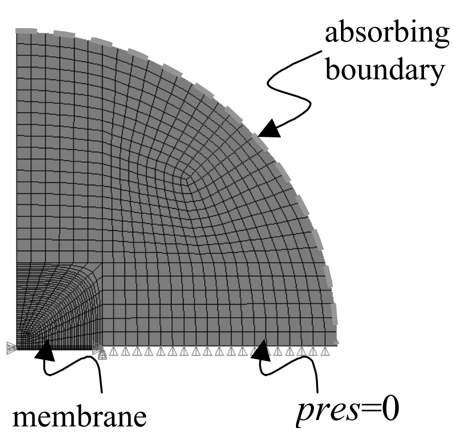

Figure 5. FEM mesh for a single membrane.

Using the above model, first we calculated the radiation impedance of a circular area assuming a rigid baffle boundary condition next to the radiating region. For this configuration, radiation impedance can be calculated analytically. Our results matched well with the analytical solution proving the validity of the FEM model. Then, we used the same model to calculate the radiation impedance for a pressure release baffle, which is more realistic boundary condition for devices operating in relatively high impedance medium such as water. According to equivalent circuit model, the membrane impedance equal to sum of this radiation impedance and the membrane impedance in vacuum. Figs. 5 and 6 show the calculation result. We also calculated the membrane impedance in water by applying a uniform force over the membrane surface in water and calculating the resulting average velocity. This result is also depicted in Figs. 5 and 6.

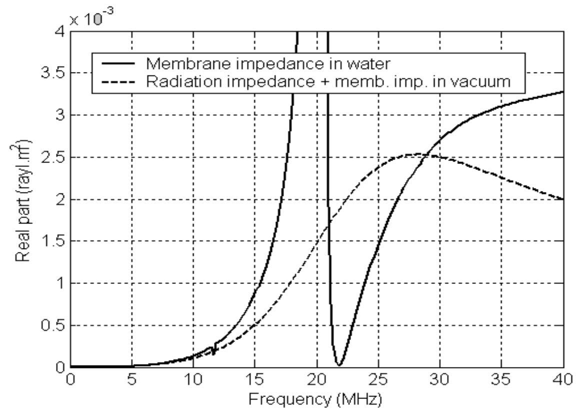

Figure 6. Real part of the membrane impedance calculated using equivalent circuit approach and FEM.

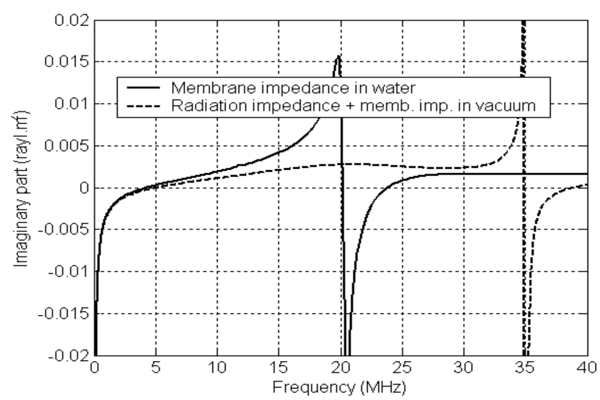

Figure 7. Imaginary part of the membrane impedance calculated using equivalent circuit approach and FEM.

From above figures it is clear that membrane impedance in water is not simply equal to the addition of the two impedances. Due to the water loading anti-resonance of the membrane shifts to a lower frequency.

C. Immersion FEM calculation (many cells)

In a CMUT transducer, many of the membranes operate in parallel and resulting radiation impedance is real. For this case we constructed a 3D FEM model. By using the similar approach used in the previous subsection, our calculations showed that addition of the membrane impedance and the radiation impedance did not provide an accurate estimate of the membrane impedance in water. In addition, we also incorporated electrostatic elements into the 3D model and compared transmit pressure to that of calculated using equivalent circuit model.

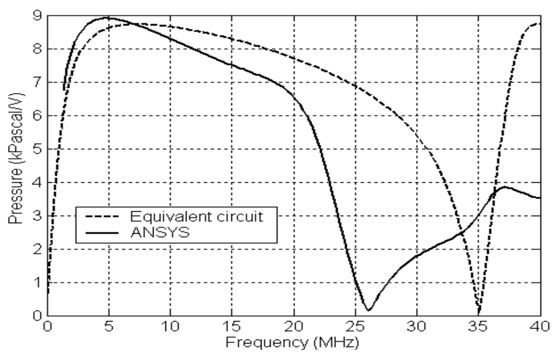

Figure 8. Transmit pressure calculated on the transducer's surface.

Above figure compares calculated transmit pressures for a transducer. The transmit bandwidth is determined by the anti-resonance which is not predicted correctly using the equivalent circuit model. Another interesting result we obtained using the 3D model is that half electrode coverage results in optimum transmit bandwidth as depicted in Fig. 9.

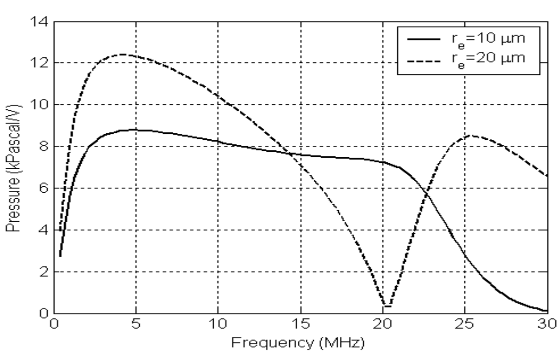

Figure 6. Transmit pressure. The pressure is calculated λ away from the transducer surface.

D. FEM of a 1D element

We have also created an FEM model for a 1D array element. The element has 7 hexagonal cells in the width dimension and it is infinitely long in the length dimension. Fig. 10 shows the output pressure. The bandwidth of the device is 12.5 MHz in transmit.

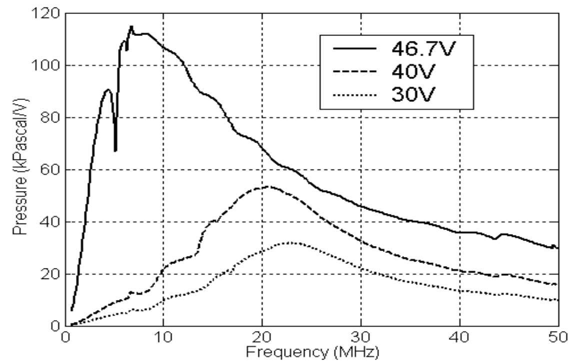

Figure 10. Transmit pressure of a 1D array.

III. Conclusion

The transmit bandwidth of a CMUT transducer is determined by the anti-resonance of the membrane. Equivalent circuit model cannot predict the anti-resonance correctly for immersion operation. In immersion the anti-resonance frequency depends on the electrode coverage on the membrane. Half electrode coverage results in maximum bandwidth since it shifts the anti-resonance to a higher frequency.

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